

Probability and Statistics Solutions – Chapter 3

Problem Set 3.1 Exercises Pages 83-86

1) $EV = 0 \cdot 0.02 + 1 \cdot 0.26 + 2 \cdot 0.37 + 3 \cdot 0.19 + 4 \cdot 0.12 + 5 \cdot 0.04 = 2.25$

2) a) Cookie Dough = $\frac{50}{75} \approx 67\%$ Butter Braids = $\frac{15}{75} = 20\%$ Bread = $\frac{10}{75} \approx 13\%$

b)

Product	Cookie Dough	Butter Braids	Bread Packs
Value	\$6	\$10	\$12
Probability	0.67	0.20	0.13

c) $EV = 6 \cdot 0.67 + 10 \cdot 0.20 + 12 \cdot 0.13 = \7.60

In other words, the expected value of a sale (average sale) was \$7.60.

Note: The table contains rounded values!

3) First build a probability model for this situation.

Value	\$0	\$5	\$10	\$30
Probability	0.30	0.40	0.25	0.05

$$EV = \$0 \cdot 0.30 + \$5 \cdot 0.40 + \$10 \cdot 0.25 + \$30 \cdot 0.05 = \$6.00$$

The owner should charge \$6.

4) First build a probability model for this situation.

Value	\$2	\$5	\$10	\$15
Probability	0.20	0.40	0.30	0.10

$$EV = \$2 \cdot 0.20 + \$5 \cdot 0.40 + \$10 \cdot 0.30 + \$15 \cdot 0.10 = \$6.90$$

The owner should charge $\$6.90 + \$1.50 = \$8.40$ for a player to play this game.

5) $EV = 1 \cdot 0.29 + 2 \cdot 0.38 + 3 \cdot 0.17 + 4 \cdot 0.11 + 5 \cdot 0.05 = 2.25$

The average racer completes 2.25 laps.

6) a) Use 'x' to represent the missing amount (???)

$$\$0 \cdot 0.32 + \$1 \cdot 0.47 + \$3 \cdot 0.08 + x \cdot 0.07 + \$21 \cdot 0.06 = \$2.53$$

$$\$0 + \$0.47 + \$0.24 + 0.07x + \$1.26 = \$2.53$$

$$0.07x = \$0.56$$

$$x = \$8.00$$

The missing payout is \$8.

b) They picked \$3 for the price instead of \$2 because the expected payout is \$2.53. If they charged only \$2, they would lose money.

c) They picked \$3 for the price instead of \$6 because \$6 is a very large amount to charge when the average payout is \$2.53. At \$6, players would quickly recognize this as a game in which you easily lose money and will be much less likely to play. Casinos don't make money unless people play.

7) $\frac{1+2+3+4+5+6}{6} = 3.5$

8)

Result	Heads	Tails
Value	\$20	\$30
Probability	0.5	0.5

$$EV = \$20 \cdot 0.5 + \$30 \cdot 0.5 = \$25$$

9) a) Use 'x' for the missing amount (???)

$$\$3 \cdot 0.25 + \$6 \cdot 0.35 + \$10 \cdot x + \$50 \cdot 0.07 = \$6.95$$

$$\$0.75 + \$2.10 + \$10x + \$3.50 = \$6.95$$

$$\$10x = \$0.60$$

$$x = 0.06$$

This is impossible because the total of the probabilities must add up to 1 and the probabilities would only add up to 0.73 if $x=0.06$.

b) We must make sure the probabilities add up to exactly 1.

$$0.25 + 0.35 + x + 0.07 = 1$$

$$x = 0.33$$

c) $EV = \$3 \cdot 0.25 + \$6 \cdot 0.35 + \$10 \cdot 0.33 + \$50 \cdot 0.07 = \$9.65$

10) a) First notice that the total of the probabilities must add up to 1.

$$0.20 + 0.25 + 0.15 + 0.30 + x = 1$$

$$x = 0.10$$

The missing probability is 10%.

b) Perform the expected value calculation. Let 'y' represent our missing prize amount.

$$\$1 \cdot 0.20 + \$3 \cdot 0.25 + \$4 \cdot 0.15 + \$7 \cdot 0.30 + y \cdot 0.10 = 4.75$$

$$\$0.20 + \$0.75 + \$0.60 + \$2.10 + 0.10y = 4.75$$

$$0.10y = \$1.10$$

$$y = \$11$$

11) a) $0 \cdot 0.03 + 1 \cdot 0.45 + 2x + 3y + 4 \cdot 0.02 = 1.6$

$$0 + 0.45 + 2x + 3y + 0.08 = 1.6$$

$$2x + 3y = 1.07$$

b) $0.03 + 0.45 + x + y + 0.02 = 1$

$$x + y = 0.50$$

c) We will now solve the system $\begin{cases} 2x + 3y = 1.07 \\ x + y = 0.50 \end{cases}$ using linear combinations (elimination).

$$\begin{array}{r} 2x + 3y = 1.07 \\ -2(x + y = 0.50) \end{array} \text{ which gives } \begin{array}{r} 2x + 3y = 1.07 \\ -2x - 2y = -1 \end{array} \text{ . When we add, the 'x' terms cancel.}$$

$$2x + 3y = 1.07$$

$$\underline{-2x - 2y = -1}$$

$$y = 0.07$$

Since $x + y = 0.50$ we can also quickly determine that 'x' must equal 0.43.

$$x = 0.43 \text{ and } y = 0.07$$

Problem Set 3.1 Review Exercises

12) a) $P(\text{Male}) = \frac{145}{325} = \frac{29}{65} \approx 0.45$

b) $P(\text{Cell}) = \frac{175}{325} = \frac{7}{13} \approx 0.54$

c) $P(\text{Male} | \text{TV}) = \frac{55}{75} = \frac{11}{15} \approx 0.73$

d) $P(\text{Computer} | \text{Female}) = \frac{45}{180} = \frac{1}{4} = 0.25$

13) There is no indication that order matters. The employee will pick out chairs **and** tables.

There are ${}_{10}C_3 \cdot {}_4C_2 = 120 \cdot 6 = 720$ ways to do this.

14) a) $P(\text{Red}) = \frac{26}{52} = \frac{1}{2} = 0.5$

b) $P(\text{Spade}) = \frac{13}{52} = \frac{1}{4} = 0.25$

c) $P(\text{Face}) = \frac{12}{52} = \frac{3}{13} \approx 0.23$

d) $P(\text{Heart} | \text{Red}) = \frac{13}{26} = \frac{1}{2} = 0.5$

Problem Set 3.2 Exercises Pages 90-93

1) Consider the probability model shown below.

Color	White	Red	Blue
Value	\$0	\$10	\$20
Probability	$\frac{25}{30}$	$\frac{4}{30}$	$\frac{1}{30}$

$$EV = \$0 \cdot \frac{25}{30} + \$10 \cdot \frac{4}{30} + \$20 \cdot \frac{1}{30} = \$2$$

This game should cost \$2 if it is to be a fair game.

2) First build a probability model for this situation.

Roll	1	2	3	4	5	6
Value	\$1	\$2	\$3	\$4	\$5	\$12
Probability	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$EV = \$1 \cdot \frac{1}{6} + \$2 \cdot \frac{1}{6} + \$3 \cdot \frac{1}{6} + \$4 \cdot \frac{1}{6} + \$5 \cdot \frac{1}{6} + \$12 \cdot \frac{1}{6} = \$4.50$$

3)

Value	57	58	59	0	61	62	63	64	65	66	67
Prob.	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$57 \cdot \frac{1}{36} + 58 \cdot \frac{2}{36} + 59 \cdot \frac{3}{36} + 0 \cdot \frac{4}{36} + 61 \cdot \frac{5}{36} + 62 \cdot \frac{6}{36} + 63 \cdot \frac{5}{36} + 64 \cdot \frac{4}{36} + 65 \cdot \frac{3}{36} + 66 \cdot \frac{2}{36} + 67 \cdot \frac{1}{36}$$

$$EV = \frac{1,992}{36} = \frac{166}{3} \approx 55.33$$

If you roll one more time, the average result is 55.33 which is better than stopping with 55 points. It is to your advantage to roll one more time.

4)

Value	62	63	64	0	66	67	68	69	70	71	72
Prob.	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$62 \cdot \frac{1}{36} + 63 \cdot \frac{2}{36} + 64 \cdot \frac{3}{36} + 0 \cdot \frac{4}{36} + 66 \cdot \frac{5}{36} + 67 \cdot \frac{6}{36} + 68 \cdot \frac{5}{36} + 69 \cdot \frac{4}{36} + 70 \cdot \frac{3}{36} + 71 \cdot \frac{2}{36} + 72 \cdot \frac{1}{36}$$

$$EV = \frac{2,152}{36} = \frac{538}{9} \approx 59.78$$

If you roll one more time, the average result is 59.78 which is worse than stopping with 60 points. It is not to your advantage to roll one more time.

- 5) Using the fact that it is advisable to roll when you have 55 points but not advisable to roll if you have accumulated 60 points, we know the 'break-even' point must be somewhere between 55 and 60. I will start with a guess of 58 points.

Value	60	61	62	0	64	65	66	67	68	69	70
Prob.	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$60 \cdot \frac{1}{36} + 61 \cdot \frac{2}{36} + 62 \cdot \frac{3}{36} + 0 \cdot \frac{4}{36} + 64 \cdot \frac{5}{36} + 65 \cdot \frac{6}{36} + 66 \cdot \frac{5}{36} + 67 \cdot \frac{4}{36} + 68 \cdot \frac{3}{36} + 69 \cdot \frac{2}{36} + 70 \cdot \frac{1}{36}$$

$$EV = \frac{2,088}{36} = 58$$

At 58 points, it makes no difference if you stop or roll once more.

- 6) In order to win, a player must get all three digits correct.

$$P(\text{All 3 Correct}) = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{1000} = 0.001$$

This implies that the chance of not winning is 0.999.

$$EV = \$500 \cdot \frac{1}{1000} + \$0 \cdot \frac{999}{1000} = \$0.50$$

The expected value is \$0.50 and since they pay \$1 to play, the average player loses \$0.50 every time they play.

- 7) In order to win, the player must pick blue and blue, red and red, or yellow and yellow.

$$P(\text{match}) = \frac{12}{30} \cdot \frac{11}{29} + \frac{10}{30} \cdot \frac{9}{29} + \frac{8}{30} \cdot \frac{7}{29} = \frac{278}{870} = \frac{139}{435} \approx 0.32$$

The expected value is \$3.83 as shown in the calculation below.

$$\$12 \cdot .32 + \$0 \cdot .68 = \$3.83$$

Since the game costs \$5 to play, the expected loss for a player is \$5 - \$3.83 = \$1.17.

- 8) The only way the insurance company loses money is if the collection is lost, stolen, or destroyed. If this happens, they have to pay out \$20,000. If this does not happen, they pay nothing.

Value	\$20,000	\$0
Probability	0.002	0.998

$$EV = \$20,000 \cdot 0.002 + \$0 \cdot 0.998 = \$40$$

The insurance company expects to pay out \$40 per year on this policy. Since they collect \$300, they would have an annual profit of \$260.

- 9) Build a probability model to organize the information. Note that the table takes into consideration the original cost of the land.

Value	\$130,000	\$90,000	\$0
Probability	0.20	0.70	0.10

$$EV = \$130,000 \cdot 0.20 + \$90,000 \cdot 0.70 + \$0 \cdot 0.10 = \$89,000$$

The expected selling price for a parcel of land is \$89,000 so the prospector made a good investment since each parcel costs \$50,000. The prospector will expect to make a profit of \$39,000 on each parcel.

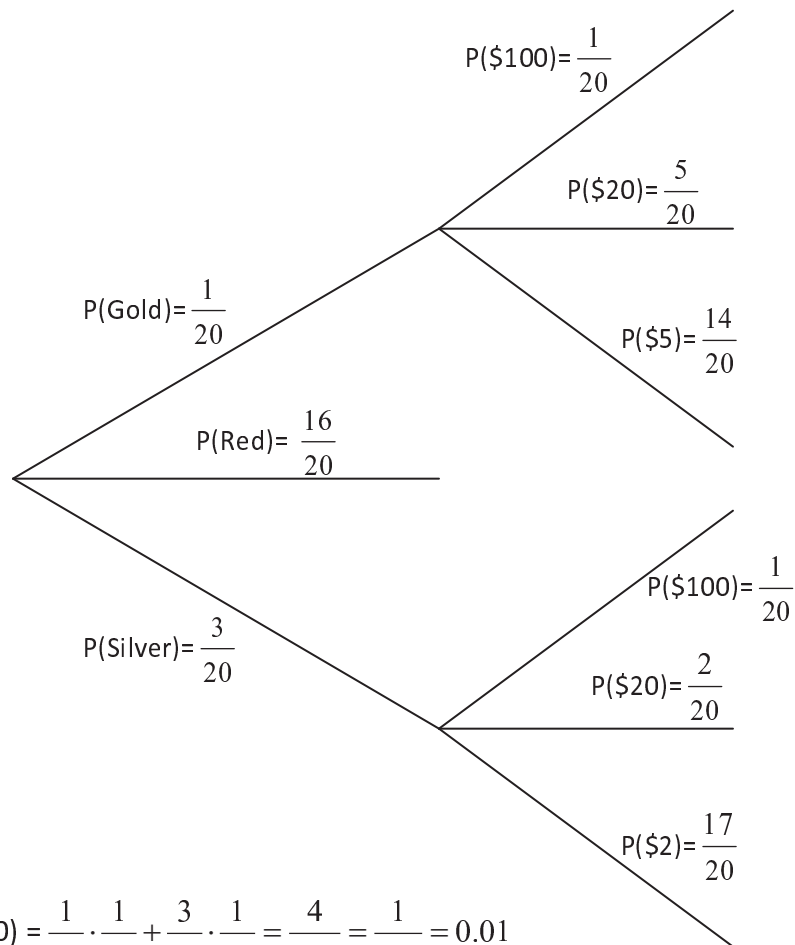
10) Begin with by building a probability model that tracks what might happen.

Value	\$250,000	\$0
Probability	0.000943	0.999057

$$EV = \$250,000 \cdot 0.000943 + \$0 \cdot 0.999057 = \$235.75$$

Since the amount collected is \$360 and the expected payout is \$235.75, the insurance company makes an average \$124.25 on this particular policy.

11) a)



$$b) P(\$100) = \frac{1}{20} \cdot \frac{1}{20} + \frac{3}{20} \cdot \frac{1}{20} = \frac{4}{400} = \frac{1}{100} = 0.01$$

$$P(\$20) = \frac{1}{20} \cdot \frac{5}{20} + \frac{3}{20} \cdot \frac{2}{20} = \frac{11}{400} \approx 0.03$$

$$P(\$5) = \frac{1}{20} \cdot \frac{14}{20} = \frac{14}{400} = \frac{7}{200} \approx 0.04$$

$$P(\$2) = \frac{3}{20} \cdot \frac{17}{20} = \frac{51}{400} \approx 0.13$$

Value	\$0	\$2	\$5	\$20	\$100
Probability	0.80	0.13	0.04	0.03	0.01

Note: The probability model uses some rounded values but the expected value calculation uses exact values.

$$c) EV = \$0 \cdot 0.80 + \$2 \cdot 0.13 + \$5 \cdot 0.04 + \$20 \cdot 0.03 + \$100 \cdot 0.01 = \$1.98$$

This represents the total payout, however, the player had to pay \$5 to play so the actual loss is $\$5.00 - \$1.98 = \$3.02$.

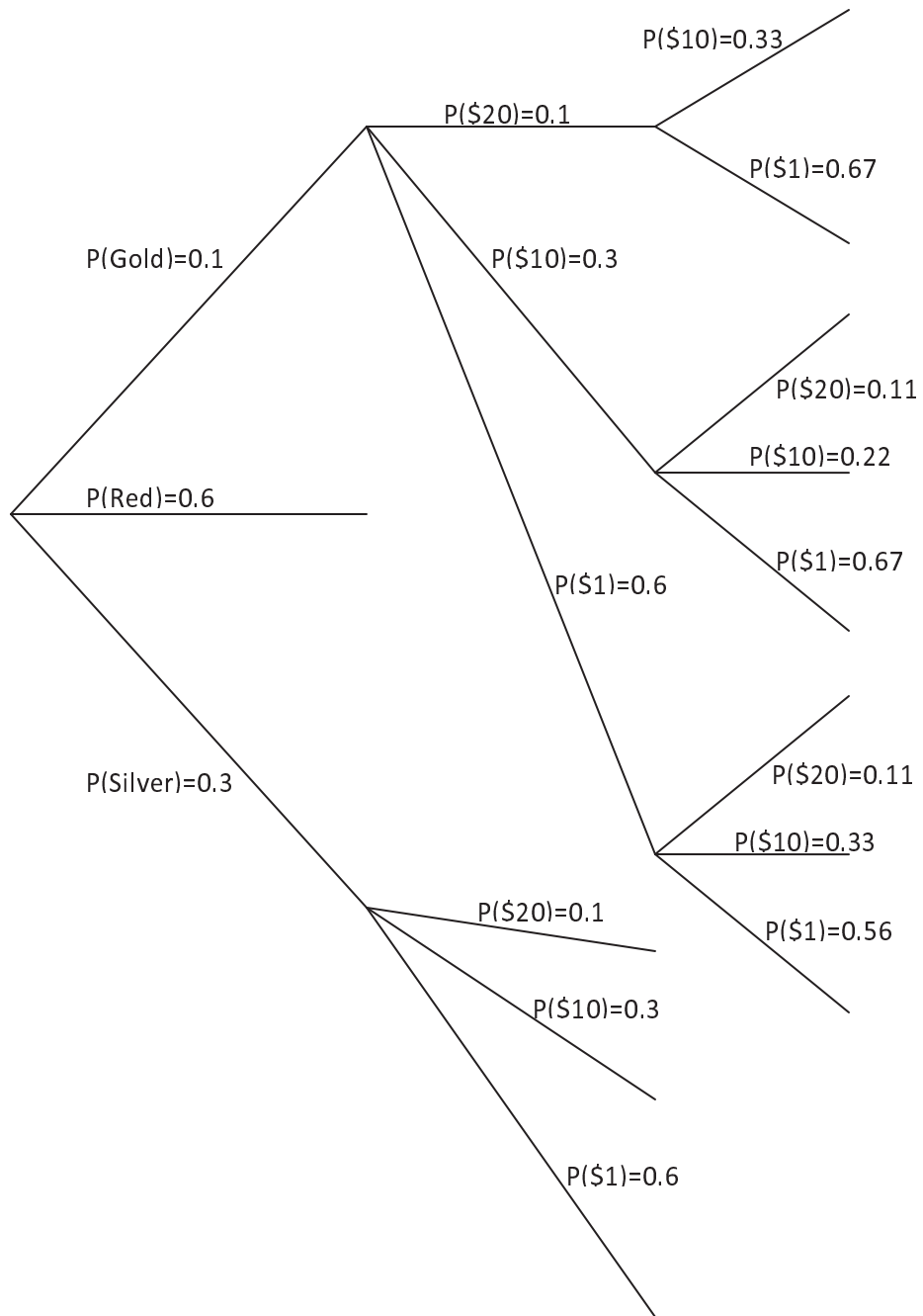
12)

Color	Red	Blue	Green	Yellow
Value	\$2	\$4	\$5	\$5
Probability	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$EV = \$2 \cdot \frac{1}{2} + \$4 \cdot \frac{1}{6} + \$5 \cdot \frac{1}{6} + \$5 \cdot \frac{1}{6} \approx \$3.33$$

This game should cost \$3.33 to be a fair game.

13) Start by building a tree diagram.



The amounts of money that may be won are \$1, \$2, \$10, \$11, \$20, \$21, & \$30.

Value	\$0	\$1	\$2	\$10	\$11	\$20	\$21	\$30
Probability	0.6	0.18	0.03	0.09	0.04	0.04	0.01	0.01

The expected value of the payout is \$2.80. Since the game costs \$3 to play, you will lose an average of \$0.20 every time you play. You should not play.

14) Start by asking how many ways we could select three digits where order doesn't matter.

Scenario #1: All three digits are different. We have 10 digits to pick from and we need to select three. This is a combination since order does not matter. ${}_{10}C_3 = 120$

Scenario #2: Two of the digits are the same. For example, we could have 001, 002, 003,.... There are nine ways to do this if we have two 0's. There will also be nine ways to do this if we have two 1's, two 2's, two threes,.... In total we have a total of $10 \cdot 9 = 90$ ways to do this.

Scenario #3: All three digits are the same (000, 111, etc...) There are only 10 ways to do this.

Adding all three scenarios, there are $120 + 90 + 10 = 220$ ways that three digits can be selected when we don't pay attention to order. Therefore, there is a $\frac{1}{220}$ chance that

all three numbers will match but not necessarily in the right order. Keep in mind that there is still a $\frac{1}{1000}$ chance that the 3 numbers will be in exactly the right order. As a

result, there is a $\frac{1}{220} - \frac{1}{1000} = \frac{39}{11,000} \approx .003$ chance that the three digits will be correct

but not in the right order.

Value	\$0	\$500	\$80
Probability	$\frac{219}{220}$	$\frac{1}{1,000}$	$\frac{39}{11,000}$

$$EV = \$0 \cdot \frac{219}{220} + \$500 \cdot \frac{1}{1,000} + \$80 \cdot \frac{39}{11,000} \approx \$0.78$$

This tells us that the average player gets \$0.78 every time they play. Since a player must pay \$1 to play, the average player will lose \$0.22.

Problem Set 3.2 Review Exercises

15) a) Since the sum of the probabilities must equal 1, 'X' = 0.55.

b) $EV = \$2 \cdot 0.55 + \$4 \cdot 0.25 + \$7 \cdot 0.15 + \$11 \cdot 0.05 = \$3.70$

16) We must select juniors **and** seniors so we find that there are ${}_8C_2 \cdot {}_8C_4 = 28 \cdot 70 = 1,960$ ways to select our 6 committee members.

17) The three cards must either be all red or all black. It does not matter what the color of the first card is, we just must make sure cards 2 and 3 match it.

$$P(\text{All Same Color}) = 1 \cdot \frac{25}{51} \cdot \frac{24}{50} = \frac{600}{2,550} = \frac{4}{17} \approx 0.24$$

18) $S = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$

Problem Set 3.3 Exercises Pages 98-100

Note: Answers may vary in this section from student answers as there are many ways that digits can be assigned from a random digits table.

- 1) a) You could assign the digits 1 to 8 to represent a student that wants to eliminate final exams and assign 9 and 0 to represent a student who does not want to eliminate final exams.
b) Using the method in part a), 19 of the 20 students selected were in support of eliminating final exams.
- 2) a) Since the probabilities must add up to 1, the missing probability must be 0.1 or 10%.
b) Assign 1 and 2 to the top 10%, 3 to 6 for the 10% to 25% group, 7 to 9 for the 25% to 50% group, and 0 for the bottom 50%.
c) The table below summarizes the results for the method in part b).

Group	Top 10%	10% - 25%	25% - 50%	Bottom 50%
# of Results	1	7	8	4

- 3) a) Since these percents are to the nearest hundredth, I will need to select two digits at a time.
Assign 01 to 20 for A's, 21 to 49 for B's, 50 to 84 for C's, 85 to 99 & 00 to D's and F's.

b) The table below summarizes the results for this simulation from line 106.

Grade	A's	B's	C's	D's or F's
# of Results	6	3	15	6
% of Results	20%	10%	50%	20%

c) The A's were right on, the B's were way low, the C's were way high, and the D's or F's were close.

4) a) Use two digits at a time using 01 to 52 to represent the cards in the deck. Ignore 53 to 99 and 00 and also ignore any repeats because it is impossible to get the exact same card two times in one hand. The chart below shows the assignment of digits.

Hearts	Digits		Diam.	Digits		Clubs	Digits		Spades	Digits
A	01		A	14		A	27		A	40
2	02		2	15		2	28		2	41
3	03		3	16		3	29		3	42
4	04		4	17		4	30		4	43
5	05		5	18		5	31		5	44
6	06		6	19		6	32		6	45
7	07		7	20		7	33		7	46
8	08		8	21		8	34		8	47
9	09		9	22		9	35		9	48
10	10		10	23		10	36		10	49
J	11		J	24		J	37		J	50
Q	12		Q	25		Q	38		Q	51
K	13		K	26		K	39		K	52

b)

Hand 1 = 16, 08, 40, 22, 15 3D, 8H, AS, 9D, 2D No Matches

Hand 2 = 47, 13, 35, 17, 28 8S, KH, 9C, 4D, 2C No Matches

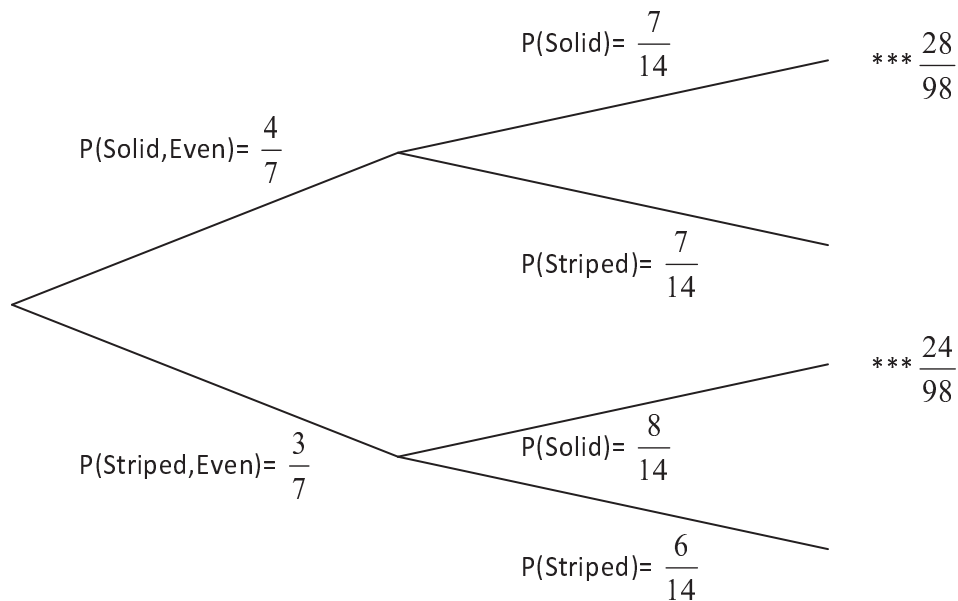
Hand 3 = 04, 10, 43, 35, 34 4H, 10H, 4S, 9C, 8C This has both the 4 of Hearts & 4 of Spades.

In this setup, it took 3 trials before there was a hand with two matching cards.

- 5) a) Even though it is unlikely to happen, four 6's in a row are possible.
 b) Roughly 10% of all digits should be nine, however there is no guarantee that exactly 10% are nine's. It is similar to flipping a coin. We might expect roughly 50% heads, but it certainly could be slightly higher or lower than that as flips are completed.
 c) Go to the next line in the table and continue your process where you left off.
- 6) a) Assign three digits at a time to represent the day of the year the student was born. In this case, use 001 through 365 and ignore three digit combinations that are greater than 365 and also ignore 000.
 b) The list below represents the 30 students selected using the method from part a).
 077, 148, 168, 347, 052, 245, 102, 290, 136, 081, 271, 025, 330, 184, 281, 350, 143, 271, 276, 119, 159, 121, 184, 103, 360, 277, 099, 330, 008, 173
 In this simulation, there were three pairs of students born on the same day of the year. These pairs of students were born on the 184th, 330th, and 271st day of the year.

Problem Set 3.3 Review Exercises

- 7) In one interpretation of this problem, there are 8 cards that would be good for the first card. Once you have that card (a king or an ace), there would only be 4 cards that would be acceptable for the second card. $P(K \& A) = \frac{8}{52} \cdot \frac{4}{51} = \frac{32}{2652} = \frac{8}{663} \approx 0.01$
- 8) a) Remember, there are only 7 even pool balls in the bag. Of those, the 2, 4, 6, and 8 balls are solid while the other 3 are striped. Building a tree diagram will be helpful here.



Only the starred branches will matter. These give $\frac{28}{98} + \frac{24}{98} = \frac{52}{98} = \frac{26}{49} \approx 0.53$.

b) We want the probability of the pool ball having an even number if we know it is striped.

There are seven striped pool balls in the bag. Of those, only 3 have even numbers. $\frac{3}{7} \approx 0.43$

9) a) $36 + 11 + 8 = 55$

b) $97 - 55 = 42$

c) $P(\text{Hockey} | \text{Football}) = \frac{11}{47} \approx 0.23$

Chapter 3 Review Exercises Pages 101-102

1) The only way to win is to get a red and red or a blue and blue.

$$P(\text{Red \& Red}) = \frac{10}{25} \cdot \frac{9}{24} = \frac{90}{600} = \frac{3}{20} = 0.15$$

$$P(\text{Blue \& Blue}) = \frac{15}{25} \cdot \frac{14}{24} = \frac{210}{600} = \frac{7}{20} = 0.35$$

Therefore, there must be a 50% chance of losing if the marbles don't match.

Result	No Match	Red & Red	Blue & Blue
Value	\$0	\$10	\$5
Probability	0.50	0.15	0.35

$$EV = \$0 \cdot 0.50 + \$10 \cdot 0.15 + \$5 \cdot 0.35 = \$3.25$$

The average payout is \$3.25 but it costs \$5 to play so I would expect to lose an average of \$1.75 every time I play. It is not to your advantage to play this game.

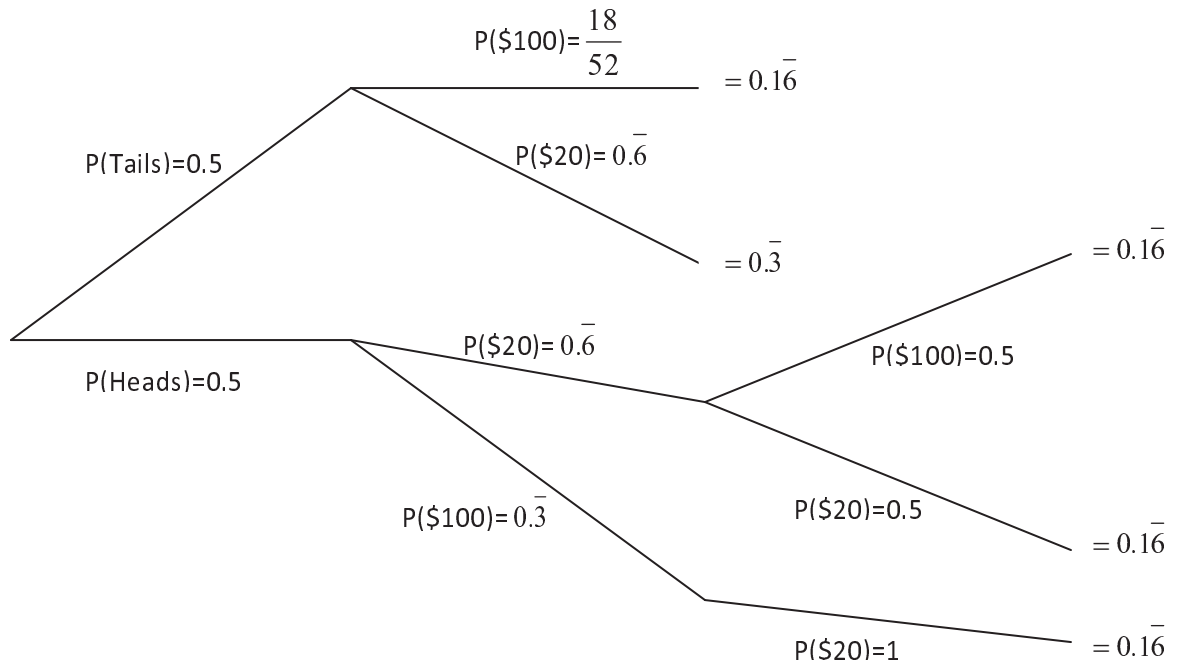
2) Use only the digits 1 to 6 and ignore all other digits. Since we are rolling two dice, we must get two results for each trial. For example, my first result is a 5 and a 5 giving a total of ten. My results are summarized in the table below.

Total	2	3	4	5	6	7	8	9	10	11	12
# of Results	1	1	2	3	6	5	6	4	4	1	2
≈ Percent	3%	3%	6%	8%	17%	14%	17%	11%	11%	3%	6%
Theoretical Percent	3%	6%	8%	11%	14%	17%	14%	11%	8%	6%	3%

The theoretical percents are based upon the dice chart below. While the results are not an exact match, a pattern emerges showing that our simulation has higher percentages in the middle range of totals and lower percentages on the lower and higher totals. This matches the theoretical percentage pattern quite well.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

3) Start by building the tree diagram.



The possible totals are \$20, \$40, \$100, and \$120. The probability model based upon the tree diagram can now be constructed. All probability values shown are rounded.

Value	\$20	\$40	\$100	\$120
Probability	0.33	0.17	0.17	0.33

The expected value is $EV = \$20 \cdot 0.33 + \$40 \cdot 0.17 + \$100 \cdot 0.17 + \$120 \cdot 0.33 = \$70$. In order for this to be a fair game it should cost \$70.

- 4) Assign each student two digits from 01 to 38. Select two digit pairs ignoring any result greater than 38 and 00 and any repeats. The four students that get selected are numbers 12, 14, 36, and 11. Note that a result of 12 came up twice but the second 12 had to be ignored or we would have picked the same student twice.
- 5) Solve this problem by building a grid that shows different results possible.

Bill			
Spin	\$1	\$5	\$10
1	\$1	\$5	\$10
2	\$2	\$10	\$20
3	\$3	\$15	\$30

Since each result is equally likely, they all have the same probability of $\frac{1}{9}$.

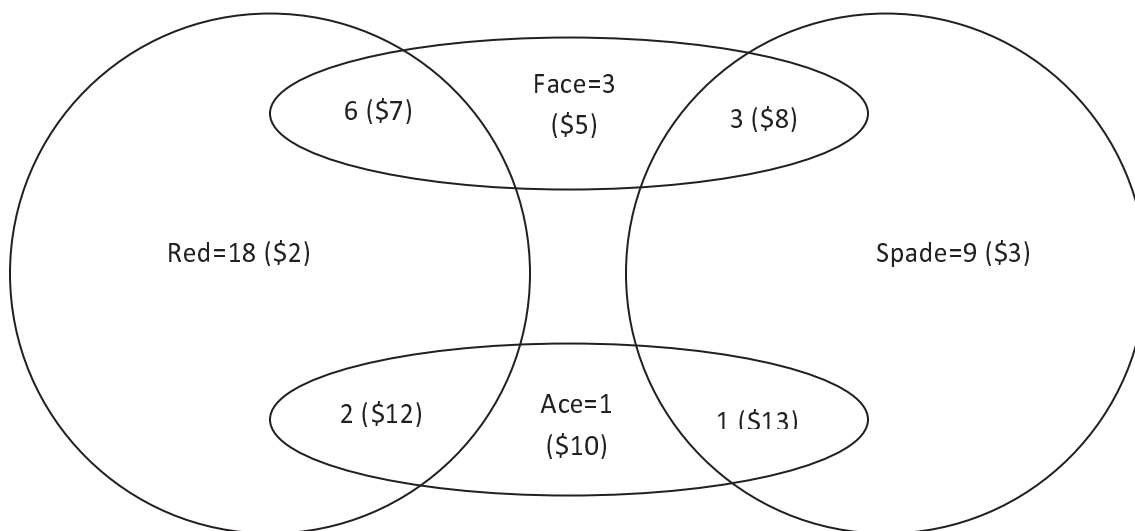
$$EV = \frac{1}{9} \cdot \$1 + \frac{1}{9} \cdot \$5 + \frac{1}{9} \cdot \$10 + \frac{1}{9} \cdot \$2 + \frac{1}{9} \cdot \$10 + \frac{1}{9} \cdot \$20 + \frac{1}{9} \cdot \$3 + \frac{1}{9} \cdot \$15 + \frac{1}{9} \cdot \$30$$

This gives an expected value of \$10.67 which is what should be charged to make this a fair game.

- 6) a) Since the probability values must add up to 1, the ??? must equal 0.16.
- b) Select two digits at a time. 'Bus' will be 01 to 31, 'Walk' will be 32 to 45, 'Car' will be 46 to 84, and 'Other' will be 85 to 99 and 00.
- c) The results of this simulation are given in the chart below.

Method	Bus	Walk	Car	Other
# of Results	4	0	3	3

- 7) a) The most likely number of centers hit is 17.
- b) $EV = 15 \cdot 0.04 + 16 \cdot 0.12 + 17 \cdot 0.35 + 18 \cdot 0.28 + 19 \cdot 0.18 + 20 \cdot 0.03 = 17.53$
The expected number of centers hit is 17.53.
- 8) One way to organize the information is by using a Venn diagram. There are 4 circles, reds, spades, faces, and aces. Note that the red circle and spade circle will not overlap and the ace circle will not overlap the face circle.



Note that this represents 43 cards from a deck. The other 9 cards (2-10 of clubs) are not worth anything.

Value	\$0	\$2	\$3	\$5	\$7	\$8	\$10	\$12	\$13
Prob.	$\frac{9}{52}$	$\frac{18}{52}$	$\frac{9}{52}$	$\frac{3}{52}$	$\frac{6}{52}$	$\frac{3}{52}$	$\frac{1}{52}$	$\frac{2}{52}$	$\frac{1}{52}$

$$EV = \$0 \cdot \frac{9}{52} + \$2 \cdot \frac{18}{52} + \$3 \cdot \frac{9}{52} + \$5 \cdot \frac{3}{52} + \$7 \cdot \frac{6}{52} + \$8 \cdot \frac{3}{52} + \$10 \cdot \frac{1}{52} + \$12 \cdot \frac{2}{52} + \$13 \cdot \frac{1}{52}$$

$$EV = \$3.67$$

This game should cost \$3.67 in order to be a fair game.